# Vertical Product Differentiation, Entry-Deterrence Strategies, and Entry Qualities

Yong-Hwan Noh and GianCarlo Moschini

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**Center for Agricultural and Rural Development**
**Iowa State University**
**Ames, Iowa 50011-1070**
***www.card.iastate.edu***

*Yong-Hwan Noh is an economist with the Bank of Korea (Seoul, Republic of Korea) and GianCarlo Moschini is a professor of economics at Iowa State University (Ames, Iowa, USA).*



Questions or comments about the contents of this paper should be directed to Yong-Hwan Noh, Institute for Monetary and Economic Research, Bank of Korea, Seoul 100-794, Republic of Korea; Ph: +82-2-759-5416; E-mail: yonghwan.noh@gmail.com.

## Abstract

We analyze the entry of a new product into a vertically differentiated market in which an entrant and an incumbent compete in prices. Here the entry-deterrence strategies of the incumbent firm rely on “limit qualities.” With a sequential choice of quality, a quality-dependent marginal production cost, and a fixed entry cost, we relate the entry-quality decision and the entry-deterrence strategies to the level of entry cost and the degree of consumer heterogeneity. Quality-dependent marginal production costs in the model entail the possibility of inferior-quality entry as well as an incumbent’s aggressive entry-deterrence strategies of increasing its quality level toward potential entry. Welfare evaluation confirms that social welfare is not necessarily improved when entry is encouraged rather than deterred.

**Keywords:** entry deterrence; quality choice; vertical product differentiation.

***JEL classification***: C72; D43; L13

## 1. Introduction

Quality choice behavior of economic agents is often studied within the “vertical product differentiation” (VPD) model, where product variants differ in their quality and consumers differ in their willingness to pay for quality, following the pioneering work of Mussa and Rosen (1978), Gabszewicz and Thisse (1979), and Shaked and Sutton (1982, 1983). A topic of interest in this context—noted earlier by Bain (1956, chapter 4) and Dixit (1979) but arguably not broadly studied—concerns an incumbent firm’s use of the product differentiation advantage as a strategy for entry deterrence. Many models of strategic entry deterrence deal with “limit pricing” or “limit quantities,” rather than “limit qualities,” as the established firm’s strategic tool for deterring entry. But clearly, as recognized by Schmalensee (1978) among others, firms can compete in non-price aspects such as product differentiation. Indeed, quality choices are of paramount importance in industries in which innovation is critical, such as in the high-technology sector. In this paper we provide a specific study of entry deterrence in the context of a VPD model.

In Dixit (1979), a representative consumer’s utility is a function of all the differentiated goods. Thus, only the degree of product differentiation matters, and no one good is superior to the others. In contrast to the formulation of Dixit (1979), Hung and Schmitt (1988, 1992) and Donnenfeld and Weber (1992, 1995) used a type of Shaked and Sutton (1982) VPD model where goods can be directly ranked by qualities to examine how the incumbent’s choice of product quality depends on the size of the entrant’s setup costs. The original VPD model of Shaked and Sutton (1982) showed that quality differences relax price competition: one firm selects the maximum product quality and the other chooses the minimum quality to lessen price competition in the production stage of the game, in the absence of an entry threat. Although entry deterrence can only be temporary, Hung and Schmitt (1988, 1992) altered this framework by introducing sequential entry and subsequent threat of entry. Thus, they showed that the threat of entry induces the incumbent firm (or the first mover) to provide a lower product quality than the technological maximum quality. Also, with the threat of entry, Hung and Schmitt showed that quality differentiation in duopoly equilibrium is reduced.

The idea of "limit quality," the minimum quality of the incumbent that deters entry, is clearly suggested by Donnenfeld and Weber (1995). They investigated how product competition among duopoly incumbents and a potential entrant's fixed entry cost affect the entry-deterrence strategies and product qualities.[1] Their result shows that rivalry among incumbents associated with simultaneous quality choice results in excessive entry deterrence, while the incumbents are likely to accommodate entry if they collude. In particular, they confirmed the result of Shaked and Sutton (1982) under the assumption of sufficiently high fixed entry costs, in that entry is blockaded and incumbents choose maximally differentiated product qualities to reduce price competition.

The foregoing VPD models are based on the assumption that there are fixed costs of quality improvement. Such fixed costs are typically dependent upon the size of the quality improvement, but the marginal cost of production itself is not affected by the quality level. The results from Hung and Schmitt (1988, 1992) and Donnenfeld and Weber (1995) are also limited to the case of quality-independent marginal costs. Thus, this setup cannot reflect the fact that the higher-quality good may be more expensive to manufacture (because of, for instance, requirements of more skilled labor or more expensive raw materials and inputs). This observation is important because, with quality-dependent production cost, the standard VPD "high-quality advantage" result (in which the firm choosing to produce the high-quality good earns higher profits in equilibrium than does the low-quality firm) need not hold.[2]

[1] A similar analysis, with two incumbents and one potential entrant, and in which both variable and fixed costs for improving qualities are zero, was presented in Donnenfeld and Weber 1992.

[2] Choi and Shin (1992), Tirole (1988), Aoki and Prusa (1996), and Lehmann-Grube (1997) impose the high-quality advantage by assuming a quality-independent production cost structure, while Lambertini (1996) suggests that the high-quality advantage with sequential or simultaneous quality choice does not necessarily hold under the assumption of quality-dependent production cost.

This study contributes to our understanding of quality choices in a VPD setting by pursuing the following three issues. First, we investigate the incumbent monopolist's strategic entry deterrence through quality choices and we examine how the level of a fixed entry cost and the degree of consumer heterogeneity affect the incumbent's choice of product quality. Second, we consider the factors that determine why an innovative entrant chooses a superior or an inferior technology compared with the existing incumbent's variety. Thus, the firm's choice of whether to be a low-quality or a high-quality provider is endogenous. We relate this issue to the entry-deterring strategies of the incumbent firm. Third, we explore the welfare implications of entry. In particular, we ask how many varieties and what quality choices of entry are socially desirable and whether entry deterrence is disadvantageous to consumers, and we evaluate market equilibrium values relative to socially optimal levels.

Specifically, this study constructs a model of a vertically differentiated product market in which both prices and product qualities are endogenous and entry is endogenous and sequential. We restrict our attention to entry-deterrence strategies within the one incumbent–one potential entrant game. Based on a Mussa and Rosen (1978) type of VPD framework, we provide a three-stage game. In stage 1, the incumbent decides its product quality. In stage 2, the potential entrant, having observed the action taken by the incumbent, decides whether to enter or not and what quality to produce in the case of entry. In the last stage of the game, both firms compete on a price level (if there is entry). Our model differs form existing related analyses (e.g., Hung and Schmitt 1988 and 1992; Donnenfeld and Weber 1992 and 1995) mostly because we specify a quality-dependent marginal production cost, such that a higher quality is associated with a higher variable cost. In addition to the fact that no particular variety guarantees higher profits under a quality-dependent marginal production cost, although firms want to differentiate products for strategic purposes (i.e., to soften price competition) they do not differentiate them completely but determine them in the interior of the feasible quality interval.[3] As in Donnenfeld and Weber (1995), we also maintain that the incumbent does not incur any entry cost, while the potential entrant must incur a fixed cost in order to enter. Entry occurs whenever strictly positive profits can be earned and can only be deterred by strategic actions of the incumbent. In particular, the incumbent acts as a Stackelberg leader in determining its product quality level.

The entry-deterrence strategies that we consider are from the pioneering idea of Bain (1956) as used in many studies (e.g., Dixit 1979; chapter 8 of Tirole 1988; and Donnenfeld and Weber 1995). According to this convention, if the fixed entry cost is large enough, the entrant would stay out of the market even if the incumbent firm ignores the possibility of entry—this is the case of "blockaded entry," whereby the incumbent monopolist does not modify its strategy and still can prevent entry. If entry is not blockaded, the incumbent has to compare the benefit of entry prevention against the cost and may either deter or accommodate entry. In the case of a "deterred entry" strategy, the incumbent modifies its behavior by increasing or decreasing quality in order to deter entry, whereas in the case of an "accommodated entry" strategy, the incumbent chooses to allow entry. In our model, therefore, the solution of the "blockaded entry" is that of an unconstrained monopolist; the solution of the "deterred entry" strategy is that of a constrained monopolist; and the solution of the "accommodated entry" strategy is that of a Stackelberg model.

We characterize how fixed entry costs and consumer heterogeneity affect the threshold conditions that describe the incumbent firm's entry-deterrence strategies (blockaded, deterred, and accommodated) and the entrant's quality choice. By introducing the quality-dependent variable costs in the model, we allow for the possibility of inferior-quality entry as well as the incumbent's entry-deterrence strategies of increasing its quality level toward a potential entry.

[3] Maximal product differentiation holds under the covered-market and quality-independent marginal production cost (e.g., Tirole 1988 and Shaked and Sutton 1982).

## 2. The Model

The analysis focuses on the entry of an innovative firm into a monopoly market. Consumers are vertically differentiated according to product qualities. Initially, there is a single established firm in an industry, the incumbent, labeled "I," who serves the entire market. A single potential entrant, labeled "E," enters the market if entry results in a positive payoff and stays out otherwise. We capture the incumbent's advantage by postulating that, whereas the entrant incurs a fixed entry cost to enter the differentiated product market, the incumbent can change its product quality without incurring this fixed cost. Assuming that the entrant needs entry costs for collecting target-market information, advertising a new product, and investing in new transportation channels, we postulate that this entry cost is invariant with respect to eventual quality levels.

The sequence of moves has three periods. In period 1, the incumbent selects its product quality $X_I$. In period 2, after observing $X_I$, the potential entrant decides whether to enter the market or not, and if entering chooses product quality $X_E$. Because entry incurs a fixed cost, a potential entrant decides to enter only if profits exceed the entry cost. If an entrant enters the market with the same quality as the existing variety, undifferentiated Bertrand competition eliminates all profits; therefore, only differentiated entry, with $X_E \neq X_I$, can be attained in equilibrium. In the last period (i.e., in the post-entry market), firms compete in prices (if the prospective entrant enters) given qualities. If the entrant stays out of the market, the incumbent behaves as a monopoly. In the case in which there is entry into the product market, the equilibrium concept that we employ is subgame perfection with Bertrand competition in the third stage of the game.

### *2.1. Costs and Demand Structure*

We modify the monopolist's quality-choice model proposed by Mussa and Rosen (1978) into the duopoly model associated with an entry game. First of all, in the second period of the game, we suppose that the quality follower (a potential entrant) is free to choose any quality level by incurring a sunk and deterministic entry cost $F > 0$.[4] That is, the entry cost is invariant with respect to eventual quality levels. As noted earlier, the quality leader (the incumbent) has a cost advantage relative to the entrant (the quality follower) in that it does not need to incur any fixed cost to determine its product quality.

Upon entrance of the new firm, the resulting duopoly supplies vertically differentiated varieties with one-dimensional qualities $X_i \in (0, \infty)$, $i = 1, 2$, with larger values of $X_i$ corresponding to higher quality ( $X_2 > X_1 > 0$ ). To avoid the uninteresting equilibrium in which only the highest possible quality yet cheapest product is produced, we postulate a quality-dependent marginal production cost, such that the higher-quality good is more expensive to manufacture. Specifically, we assume that both firms employ the same technology in which costs of producing $Q_i$ units of quality $X_i$ are

$$C(X_i, Q_i) = X_i^2 Q_i, \tag{1}$$

where $Q_i$ is the quantity produced by a firm $i$. Note that this variable costs are strictly convex in quality, such that $C'(X_i) > 0$ and $C''(X_i) > 0$ hold, but for given quality we have a constant unit production cost. This specification of VPD, in which firms compete in prices and incur variable costs of quality, is compatible with that of some earlier models.[5] In our model, when fixed costs are either absent or quality-independent, convexity in quality of the variable cost function ensures interior solutions in the quality-choosing stage of the game.

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[4] Of course, with free entry ( $F = 0$ ), the game degenerates into a pure Stackelberg model.

[5] With two-stage quality-price or quality-quantity VPD models, Bonanno and Haworth (1998) introduced a quality-dependent linear form of marginal cost; Mussa and Rosen (1978) and Part III of Motta (1993) used quality-dependent quadratic forms of marginal cost. Thus, in this case, the quality-dependent marginal cost enters directly into the competitor's pricing strategy. Importantly, although they did not explicitly indicate it, the "high-quality advantage" does not necessarily hold in that case.

On the demand side of the market, a continuum of potential consumers is differentiated by the non-negative, one-dimensional taste parameter $\theta$. The parameter $\theta$ is assumed to be distributed uniformly with density $\delta > 0$ over an interval $[\underline{\theta},\ \overline{\theta}]$, with $\overline{\theta} > \underline{\theta} > 0$.[6] We normalize the indices as $\delta = 1$ and $\overline{\theta} - \underline{\theta} = 1$. When entry takes place, we have a situation with two goods differentiated by a quality index $X_i \in (0,\ \infty)$, $i = 1, 2$, that is observable to all. As in Mussa and Rosen (1978), the indirect utility function of a consumer $\theta$ patronizing good *i* is

$$V_i\ (P_i, X_i, \theta) = \theta X_i - P_i, \tag{2}$$

where $P_i$ and $X_i$ for $i=\{1, 2\}$ are, respectively, the price and quality variables. Thus, consumers agree on the ranking of the two goods but differ in their taste parameter $\theta$. In this setting, the consumer buys the good that provides highest surplus (or buys nothing if $V_i < 0$ for two goods).

We focus on the "covered market" outcome, where all consumers purchase a unit of the good. For given prices $(P_1,\ P_2)$, the covered market demand systems incorporating the possibility of the preempted market case are

$$Q_1 = \max\left\{0,\ \min\left\{\overline{\theta},\ \theta_{12}\right\} - \underline{\theta}\right\} \tag{3.1}$$

$$Q_2 = \max\left\{0,\ \overline{\theta} - \max\left\{\theta_{12},\ \underline{\theta}\right\}\right\},\ \text{where}\ \theta_{12} = \frac{P_2 - P_1}{X_2 - X_1}. \tag{3.2}$$

Therefore, covered-market equilibrium can be characterized by the cases in which only the high-quality good is sold, only the low-quality good is sold, or both types of goods are present in the market. In particular, for the cases in which both goods are present, the aggregate demand functions reflect a net substitution pattern (i.e., the cross-price effect is positive). In the analysis that follows, given qualities $X_1$ and $X_2$, we focus on interior solutions in which both goods are consumed in the market and all consumers are served in equilibrium.

[6] In the related but alternative formulation of Shaked and Sutton (1982, 1983), consumers differ by their incomes rather than by their tastes, and $\underline{\theta} > 0$ is equivalent to the condition that all consumers have a strictly positive income.

### *2.2. Product Market Equilibrium*

*Monopoly Market Equilibrium*

If entry does not occur, the incumbent is a monopoly. Because consumers are passive about market coverage, a monopolist determines endogenously a covered or uncovered market. Thus, to invoke the assumption of full market coverage, we need to find the parameter restriction in which the monopolist would elect to cover the market. Let us denote $\hat{\theta} \equiv P_{IM} / X_{IM}$ (where the subscript "IM" stands for the "incumbent monopoly") as the marginal consumer who is indifferent between buying a good and not buying at all. Then the market demand for a monopoly is $Q_{IM} = \overline{\theta} - \hat{\theta}$. Using the monopolist's unit cost $X_{IM}^2$, the incumbent as a monopolist in the market solves the following maximization problem with respect to price for a given quality:

$$Max\ \pi_{IM} = \left(P_{IM} - X_{IM}^2\right) Q_{IM} = \left(P_{IM} - X_{IM}^2\right)\left(\overline{\theta} - \frac{P_{IM}}{X_{IM}}\right). \tag{4}$$

From the first-order condition for this maximization problem, we know that $\partial \pi_{IM} / \partial P_{IM} \leq 0$. If we have an interior solution so that $P_{IM}^* / X_{IM} > \underline{\theta}$, the market is uncovered. But in the case of a corner solution where $P_{IM}^* / X_{IM} = \underline{\theta}$, the market is covered. For the corner solution, it is necessary that

$$\left.\frac{\partial \pi_{IM}}{\partial P_{IM}}\right|_{P_{IM} = \underline{\theta} X_{IM}} = \underline{\theta} + 1 + X_{IM} - 2\underline{\theta} \leq 0 \quad \Leftrightarrow \quad \underline{\theta} \geq 1 + X_{IM}. \tag{5}$$

In this covered-market case, the monopolist's price is at the level at which the least-value consumer ($\underline{\theta}$) gives up all its surplus to purchase the good (i.e., $P_{IM}^* = \underline{\theta} X_{IM}$). Thus, the monopolist's product market equilibrium profit is

$$\pi_{IM}^* = \underline{\theta} X_{IM} - X_{IM}^2. \tag{6}$$

*Duopoly Market Equilibrium*

Now, consider the duopoly covered market equilibrium in which duopoly firms move simultaneously in the production stage with Bertrand competition. In this stage of the game, qualities are exogenous. Then each firm maximizes its profit with respect to its own price for any given quality choice and an opponent firm's price. The profit functions of the low-quality firm and of the high-quality firm are, respectively,

$$\pi_1 = \left(P_1 - X_1^{\,2}\right)\left(\frac{P_2 - P_1}{X_2 - X_1} - \underline{\theta}\right)$$

$$\pi_2 = \left(P_2 - X_2^{\,2}\right)\left(\underline{\theta} + 1 - \frac{P_2 - P_1}{X_2 - X_1}\right),$$

so that the production stage equilibrium values (i.e., the payoffs for the quality game) are[7]

$$\pi_1^* = \left(X_2 - X_1\right)\frac{\left\{\left(X_2 + X_1\right) + \left(1 - \underline{\theta}\right)\right\}^2}{9} \tag{7}$$

$$\pi_2^* = \left(X_2 - X_1\right)\frac{\left\{-\left(X_2 + X_1\right) + \left(2 + \underline{\theta}\right)\right\}^2}{9}, \tag{8}$$

with the equilibrium prices given by

$$P_1^* = \frac{1}{3}\left\{(2X_1^{\,2} + X_2^{\,2}) + (1 - \underline{\theta})(X_2 - X_1)\right\}$$

$$P_2^* = \frac{1}{3}\left\{(X_1^{\,2} + 2X_2^{\,2}) + (2 + \underline{\theta})(X_2 - X_1)\right\}.$$

Of course, these solutions only apply when, in equilibrium, the market is in fact covered. Thus, it remains to check the following two conditions. First, for exogenously given qualities, it is necessary that

$$X_1 + X_2 - 2 \le \underline{\theta} \le X_1 + X_2 + 1. \tag{9}$$

[7] Note that $\pi_1^*$ is the incumbent's payoff and $\pi_2^*$ is the entrant's payoff when entry occurs with a superior-quality good compared with the incumbent's quality. If the entrant chooses an inferior quality then the entrant's payoff is $\pi_1^*$ and the incumbent's payoff is $\pi_2^*$.

This condition ensures non-negative demands at the duopoly product market equilibrium (i.e., $Q_1^* \geq 0$ and $Q_2^* \geq 0$). As illustrated in Figure 1, the firm producing a low-quality good would become a monopoly for extremely high consumer heterogeneity (such that $\underline{\theta} < X_1 + X_2 - 2$), whereas the firm producing a high-quality good would become a monopoly for very low consumer heterogeneity (such that $\underline{\theta} > X_1 + X_2 + 1$).[8] Thus, the restriction in (9) excludes these two extreme cases. Second, for the market to be covered, it must be the case that the consumer with the lowest marginal willingness to pay for quality ($\underline{\theta}$) has a non-negative surplus when she buys one unit of the low-quality product (i.e., $\underline{\theta}X_1 - P_1^* \geq 0$). It is verified that the following parameter restriction guarantees that each consumer buys one of the two varieties in the non-cooperative equilibrium:

$$\underline{\theta} \geq \frac{\left(2X_1^{\ 2} + X_2^{\ 2}\right) + \left(X_2 - X_1\right)}{2X_1 + X_2}. \tag{10}$$

**3. Equilibrium Quality Choices**

In this section we solve the quality stage of the game (periods 1 and 2), given the Bertrand-competition solutions at the production stage. We endogenize the entrant's choice of whether to be the low-quality or the high-quality provider, relative to the existing variety produced by the incumbent. This is a Stackelberg model of quality choices in which the leader is the incumbent firm (I) and the follower is the entrant firm (E).

[8] Heterogeneity, measured here by the ratio $\overline{\theta} / \underline{\theta}$, decreases with $\underline{\theta}$ (recall that $\overline{\theta} \equiv \underline{\theta} + 1$): the greater is $\underline{\theta}$, the more homogenous are consumers. Thus, the market is likely to be preempted by the low-quality firm when consumers are relatively heterogeneous, whereas the market is likely to be preempted by the high-quality firm when consumers are relatively homogenous.

### *3.1. Best-Response Function of the Entrant*

Consider first the case of entry with a superior quality. The entrant's reduced-form payoff function from price competition in the production stage of the game is given by equation (8), and the incumbent's payoff is given by equation (7). In period 2, a firm $E$ (the Stackelberg follower) chooses $X_E$ to maximize $\left[\pi_E^*(X_I, X_E) - F\right]$ for given $X_I$. If firm $E$ enters, its best response in terms of the incumbent's quality is given by $X_E = (X_I + \underline{\theta} + 2)/3$. Then the entrant's payoff conditional on choosing high-quality entry is given by

$$\pi_E^H(X_I, F) \equiv \pi_E^*\left(X_I, \frac{X_I}{3} + \frac{\underline{\theta}+2}{3}\right) - F = \frac{4}{9}\left(\frac{\underline{\theta}+2-2X_I}{3}\right)^3 - F. \tag{11}$$

The potential entrant enters the market only if this leads to a strictly positive payoff,[9] that is, when

$$X_I < \lambda_H \text{, where } \lambda_H \equiv 1 + \frac{\underline{\theta}}{2} - \left(\frac{3}{2}\right)^{5/3} F^{1/3}. \tag{12}$$

Now consider the case of entry with an inferior quality. The entrant's payoff function from price competition in the third stage of the game is given by equation (7), and the incumbent's payoff is given by equation (8). In this case, if firm $E$ enters, its best response in terms of the incumbent's quality is given by $X_E = (X_I + \underline{\theta} - 1)/3$. Then the entrant's payoff, conditional on choosing low-quality entry, is

$$\pi_E^L(X_I, F) \equiv \pi_E^*\left(X_I, \frac{X_I}{3} + \frac{\underline{\theta}-1}{3}\right) - F = \frac{4}{9}\left(\frac{1-\underline{\theta}+2X_I}{3}\right)^3 - F. \tag{13}$$

The potential entrant enters the market only if this leads to a positive payoff, and this holds for

$$X_I > \lambda_L \text{, where } \lambda_L \equiv \frac{\underline{\theta}-1}{2} + \left(\frac{3}{2}\right)^{5/3} F^{1/3}. \tag{14}$$

Based on these two conditional responses, we can characterize the actual "best-response function of the prospective entrant" (BRE) on the ranges of fixed costs. Let us define the critical value $\hat{X}_I(\underline{\theta}) \equiv \underline{\theta}/2 + 1/4$ such that the following equality is satisfied: $\pi_E^L(\hat{X}_I, F) = \pi_E^H(\hat{X}_I, F)$. If $X_I \leq \hat{X}_I$ then

[9] Actually, when profits are zero, the prospective entrant's choices are indifferent between entry and no entry. Here we follow the convention that the entrant enters the market only if it can make positive payoffs.

there can be superior-quality entry because $\pi_E^H \geq \pi_E^L$. Likewise, if $X_I \geq \hat{X}_I$ then there can be inferior-quality entry because $\pi_E^L \geq \pi_E^H$. Now, to define completely the BRE, we check the ranges of fixed costs. If $\lambda_L < \hat{X}_I < \lambda_H$ then the entrant's positive-profit conditions (12) and (14) are not binding. This is the case when $F < 1/18$. Whereas, if $\lambda_H < \hat{X}_I < \lambda_L$ then equations (12) and (14) are binding conditions. This holds for $F > 1/18$. For $F = 1/18$ and $X_I = \hat{X}_I$, entry does not occur because the entrant cannot make positive payoffs. Therefore, there is a discontinuity in the BRE, and we can define it on the ranges of fixed costs as follows:

$$\text{For } F < \frac{1}{18},\ X_E = \begin{cases} \dfrac{X_I}{3} + \dfrac{\underline{\theta}+2}{3}, & \text{if } X_I \leq \hat{X}_I \\ \dfrac{X_I}{3} + \dfrac{\underline{\theta}-1}{3}, & \text{if } X_I \geq \hat{X}_I \end{cases} \tag{15.1}$$

$$\text{For } F > \frac{1}{18},\ X_E = \begin{cases} \dfrac{X_I}{3} + \dfrac{\underline{\theta}+2}{3}, & \text{if } X_I < \lambda_H \\ \dfrac{X_I}{3} + \dfrac{\underline{\theta}-1}{3}, & \text{if } X_I > \lambda_L \end{cases} \tag{15.2}$$

$$\text{For } F = \frac{1}{18},\ X_E = \begin{cases} \dfrac{X_I}{3} + \dfrac{\underline{\theta}+2}{3}, & \text{if } X_I < \hat{X}_I \\ \text{No entry}, & \text{if } X_I = \hat{X}_I \\ \dfrac{X_I}{3} + \dfrac{\underline{\theta}-1}{3}, & \text{if } X_I > \hat{X}_I \end{cases} \tag{15.3}$$

where $\hat{X}_I \equiv \dfrac{\underline{\theta}}{2} + \dfrac{1}{4}$, $\lambda_H \equiv 1 + \dfrac{\underline{\theta}}{2} - \left(\dfrac{3}{2}\right)^{5/3} F^{1/3}$, and $\lambda_L \equiv \dfrac{\underline{\theta}-1}{2} + \left(\dfrac{3}{2}\right)^{5/3} F^{1/3}$.

Figure 2 shows BRE when the entrant's positive-payoff conditions (12) and (14) are not binding because fixed costs are small such that $F < 1/18$. Note that, in the quality-choice games, payoffs are zero when qualities are identical (i.e., payoffs are zero on the 45° line). Hence the BRE is necessarily discontinuous. Conditional on choosing superior-quality entry, the best response of the entrant is $\overline{ac}$ because $\pi_E^H(X_I, F) > 0$ if $X_I < \underline{\theta}/2 + 1$. Likewise, conditional on choosing inferior-quality entry, the best response of the entrant is $\overline{df}$ because $\pi_E^L(X_I, F) > 0$ if $X_I > (\underline{\theta}-1)/2$. Now, we know that

$\pi_E^L(X_I, F) \underset{<}{\overset{>}{-}} \pi_E^H(X_I, F)$ if $X_I \underset{<}{\overset{>}{-}} \hat{X}_I$. Therefore, the actual BRE when $F < 1/18$ is $\overline{abef}$ with discontinuity at $X_I = \hat{X}_I$.

The BRE when $F = 1/18$ is presented in Figure 3. In this case, BRE jumps down at $\hat{X}_I$ because the entrant's payoff $\pi_E{}^H = \pi_E{}^L = 0$ at this level of incumbent's quality.

The BRE associated with high fixed costs such that $F > 1/18$ is depicted in Figure 4. In this case, the positive-payoff conditions (12) and (14) are binding. The location of $\lambda_H$ and $\lambda_L$ depends on the size of the entrant's fixed cost. In fact, the distance between $\lambda_H$ and $\lambda_L$ increases as $F$ increases. The model thus allows the possibility of strategic behavior on the part of the incumbent. The quality leader (the incumbent), by choosing limit qualities at which the potential entrant prefers to stay out of the market, can deter entry.

### *3.2. Quality Leadership and Limit Qualities*

Consider now the strategic behavior of the incumbent at its quality stage of the game. We classify the outcomes of the incumbent's quality as a means of limiting the prospective entrant's choices. Because of discontinuity in the prospective entrant's best-response function, it is the size of the fixed cost that determines whether or not an entry-deterrence strategy is preferred.

*Parameter Restrictions on Market Outcomes*

Prior to proceeding with the analysis, it is important to note that the analysis applies only to the range of the parameter $\underline{\theta}$, which ensures that the duopoly actually covers the market. Let us first confine our attention to the post-entry duopoly (say, the case of $F < 1/18$). When entry occurs with a superior quality, BRE is given by $X_E = (X_I + \underline{\theta} + 2)/3$. The incumbent's quality choice is given by solving $\partial \pi_I^*(X_I, (X_I + \underline{\theta} + 2)/3)/\partial X_I = 0$. Accordingly, the Stackelberg solution is characterized by

$X_{IS}^{AEH} = \underline{\theta}/2 + 1/4$, $X_{ES}^{AEH} = \underline{\theta}/2 + 3/4$, $\pi_{IS}^{AEH} = 2/9$, and $\pi_{ES}^{AEH} = 1/18 - F$. In order for the duopoly market to be fully covered at the Stackelberg equilibrium, one must check whether these solutions satisfy constraints (9) and (10). Straightforward calculation shows that when entry occurs with a superior quality, the condition $\underline{\theta} \geq \sqrt{19/12} \approx 1.2583$ must be satisfied in order for both qualities to be positive and the market to be covered in equilibrium.

Next, consider the case of entry with an inferior quality. In this case, BRE is given by $X_E = (X_I + \underline{\theta} - 1)/3$. The incumbent's quality choice is then given by solving $\partial \pi_I^* \left( X_I, (X_I + \underline{\theta} - 1)/3 \right) / \partial X_I = 0$. Accordingly, the Stackelberg solution is characterized by $X_{IS}^{AEL} = \underline{\theta}/2 + 1/4$, $X_{ES}^{AEL} = \underline{\theta}/2 - 1/4$, $\pi_{IS}^{AEL} = 2/9$, and $\pi_{ES}^{AEL} = 1/18 - F$. Again, in order for the duopoly market to be covered in the Stackelberg equilibrium, one must check whether these solutions satisfy constraints (9) and (10). Straightforward calculation shows that when entry occurs with an inferior quality, the condition $\underline{\theta} \geq \sqrt{11/12} \approx 0.95743$ must be satisfied in order for both qualities to be positive and the market to be covered in equilibrium.

Consider now the pure monopoly market equilibrium, in which entry does not occur. Because consumers are passive about the market coverage, a monopolist determines endogenously a covered or uncovered market. Thus, for the specific market outcomes, we need to find the parameter restriction in which the monopolist would cover or uncover the market. As was discussed earlier, maximizing equation (6) with respect to its quality level yields a pure monopoly solution under the covered-market configuration: $X_{IM}^* = \underline{\theta}/2$ and $\pi_{IM}^* = \underline{\theta}^2/4$. For this monopoly market to be covered, this solution must satisfy the monopolist's covered-market restriction (5). Straightforward calculation shows that the condition $\underline{\theta} \geq 1 + X_{IM}^* \Leftrightarrow \underline{\theta} \geq 2$ must be satisfied in order for the monopolist's equilibrium to cover the market. Thus, for $\underline{\theta} \leq 2$, the uncovered market monopoly maximizes

$\pi_{IM}^{*} = \left(P_{IM} - X_{IM}^{2}\right)\left(\underline{\theta} + 1 - P_{IM}/X_{IM}\right)$ with respect to $P_{IM}$ and $X_{IM}$. Then the resulting equilibrium values are $P_{IM}^{*} = 2\left(1+\underline{\theta}\right)^{2}/9$, $X_{IM}^{*} = (1+\underline{\theta})/3$, and $\pi_{IM}^{*} = \left((1+\underline{\theta})/3\right)^{3}$.

In conclusion, our analysis (which is confined to the duopoly covered market case) pertains to markets with $\underline{\theta} \in \left[\sqrt{19/12}, \infty\right)$. Then, as illustrated in Figure 5, the incumbent's outcomes can be specified for two different levels of consumer heterogeneity. One is associated with the uncovered pure monopoly equilibrium in which there are relatively heterogeneous consumers such that $\underline{\theta} \in \left[\sqrt{19/12}, 2\right]$. The other is associated with the covered pure monopoly equilibrium in which there are relatively homogenous consumers such that $\underline{\theta} \geq 2$.

*Case 1: Low Fixed Costs and Accommodated Entry*

When the entry cost is sufficiently low such that $F < 1/18$, entry deterrence is not possible, so the solutions for the entry accommodation are Stackelberg duopoly equilibria. Note that if entry takes place, the duopoly firm's Stackelberg payoffs are the same regardless of which of the two possible equilibria applies. Specifically, the entrant is indifferent between entry with an inferior quality and entry with a superior quality. That is, points "b" and "e" in Figure 2 are both Stackelberg equilibria.

*Case 2: High Fixed Costs and Blockaded Entry*

If $F$ is so large that $\pi_{E}^{H} < 0$ and $\pi_{E}^{L} < 0$, the entrant cannot cover the fixed cost. That is, entry does not occur if the quality leader chooses its quality level between $\lambda_H$ and $\lambda_L$ in Figure 4. Consider first the range of relatively homogenous consumers in which $\underline{\theta} \geq 2$. When the entry cost is sufficiently large to satisfy the covered monopolist's quality level $X_{IM}^{*} = \underline{\theta}/2 \geq \lambda_H$, or equivalently $F \geq \left(2/3\right)^{5} \approx 0.13169$, the unconstrained monopoly optimum can be achieved. Thus, the entrant will not enter the market even when the incumbent plays its pure monopoly quality level. In this case, we say that entry is "blockaded."

Now, for the range of relatively heterogeneous consumers in which $\underline{\theta} \in \left[\sqrt{19/12}\,, 2\right]$, entry is blockaded if the uncovered monopolist's quality satisfies $X_{IM}^{*} = (1+\underline{\theta})/3 \geq \lambda_H$, or equivalently $F \geq \hat{F}$ where $\hat{F} \equiv (2/81)^2 (4+\underline{\theta})^3$.

*Case 3: Moderate Fixed Costs and Deterred Entry*

If $F$ falls below the boundary given by $(2/3)^5 \approx 0.13169$ for $\underline{\theta} \geq 2$, or $\hat{F}$ for $\underline{\theta} \in \left[\sqrt{19/12}\,, 2\right]$, the fixed cost of entry is insufficient to deter entry when the incumbent produces the pure monopoly quality. Then the incumbent has two choices: it could expand its quality level above the unconstrained profit-maximizing level to deter entry; or it could invite entry by choosing its quality level at a point less than $\lambda_H$ or greater than $\lambda_L$, so that entry occurs immediately and the entrant's quality level rises instantaneously to the duopoly level. To analyze the entry-deterrence strategy of the incumbent, we define $X_I^B$ as the quality level that discourages entry, where the superscript B stands for "barrier." Then $X_I^B$ is given by $\underset{X_E}{Max}\ \pi_E^{*}\left(X_E,\ X_I^B\right) - F = 0$. Thus, the incumbent can choose any quality levels in $X_I^B \in \left[\lambda_H,\ \lambda_L\right]$ to deter entry.

First, consider the case in which $F = 1/18$. If entry is accommodated, $X_I \rightarrow \hat{X}_I$ is the profit-maximizing level of quality, so that the maximum payoff that the incumbent can get from the accommodation of entry is $\lim_{X_I \rightarrow \hat{X}_I} \pi_I^{*}\left(X_I, X_E\left(X_I\right)\right) = 2/9$. Assuming that the market is covered, the incumbent's profit associated with the deterred entry is $\pi_{IM}^{*}\left(X_I^B = \hat{X}_I\right) = (2\underline{\theta}-1)(2\underline{\theta}+1)/16$. Upon checking whether this solution satisfies the monopolist's covered-market restriction (5), we find that the condition $\underline{\theta} \geq 1 + X_I^B \Leftrightarrow \underline{\theta} \geq 5/2$ must be satisfied in order for this constrained monopolist's equilibrium to cover the market. Then we know that, when $F = 1/18$, the incumbent finds it most profitable to deter entry if $\pi_{IM}^*\left(X_I^B = \hat{X}_I\right) > \lim_{X_I \to \hat{X}_I} \pi_I^*\left(X_I, X_E\left(X_I\right)\right)$. For $\underline{\theta} \geq 5/2$ in which the constrained monopoly market is covered, we have

$$K \equiv \pi_{IM}^*\left(X_I^B = \hat{X}_I\right) - \lim_{X_I \to \hat{X}_I} \pi_I^*\left(X_I, X_E\left(X_I\right)\right) = \frac{36\underline{\theta}^2 - 41}{144} > 0. \qquad (16)$$

Thus, entry is deterred by the incumbent. Now, for $\underline{\theta} \in \left[\sqrt{19/12}, 5/2\right]$, the uncovered monopolist's profit is $\pi_{IM}^*\left(X_I^B = \hat{X}_I\right) = \left(\hat{X}_I/4\right)\left(\underline{\theta} + 1 - \hat{X}_I\right)^2 = (2\underline{\theta}+1)(2\underline{\theta}+3)^2/256$ so that

$$T \equiv \pi_{IM}^*\left(X_I^B = \hat{X}_I\right) - \lim_{X_I \to \hat{X}_I} \pi_I^*\left(X_I, X_E\left(X_I\right)\right) > 0 \qquad (17)$$

because the minimum $\pi_{IM}^*$ at the lower bound of $\underline{\theta}$ is greater than the payoff from the accommodation, $\pi_{IM}^*\left(X_I^B = \hat{X}_I, \underline{\theta} = \sqrt{19/12}\right) \approx 0.418 \;>\; 2/9 \approx 0.222$. Thus, when $F = 1/18$, entry is deterred by the incumbent.

Second, consider the case in which $F > 1/18$ but entry is not blockaded. Assuming that the market is covered, because $\partial \pi_{IM}^* / \partial X_{IM} = \underline{\theta} - 2X_{IM} < 0$ for all $X_I > \underline{\theta}/2$, $X_I^B = \lambda_H$ would be the incumbent's choice when it decides to deter entry. Note that this constrained monopoly choice requires the condition $\underline{\theta} \geq 1 + X_I^B \Leftrightarrow \underline{\theta} \geq 4 - 2(3/2)^{5/3} F^{1/3}$ to cover the market. Now, we know that if entry occurs with a high quality,

$$\frac{\partial \pi_I^*\left(X_I, X_E\left(X_I\right)\right)}{\partial X_I} = \frac{2}{81}\left(4X_I + 5 - 2\underline{\theta}\right)\left(-4X_I + 1 + 2\underline{\theta}\right) > 0 \text{ for all } X_I \in \left(\frac{\underline{\theta}}{2} - \frac{1}{2},\ \hat{X}_I\right).$$

Thus, $X_I \to \lambda_H$ is the profit-maximizing level of quality if entry is accommodated, so that the maximum payoff that the incumbent can get from the accommodation of entry is $\lim_{X_I \to \lambda_H} \pi_I^*\left(X_I, X_E\left(X_I\right)\right)$. Then the incumbent finds it most profitable to deter entry if $\pi_{IM}^{*}\left(X_I^B=\lambda_H\right)>\lim\limits_{X_I\to\lambda_H}\pi_I^{*}\left(X_I, X_E\left(X_I\right)\right)$, or equivalently,

$$G\equiv 3^5\left(\underline{\theta}\lambda_H-\lambda_H{}^2\right)+\left(2\lambda_H-2-\underline{\theta}\right)\left(4\lambda_H+5-2\underline{\theta}\right)^2>0 \tag{18}$$

Because it is readily verified that the inequality in (18) holds when $\underline{\theta}\geq 2$, the incumbent will deter entry.

Third, consider the case in which $\underline{\theta}\in\left[\sqrt{19/12},\ 4-2(3/2)^{5/3}F^{1/3}\right]$ and $F>1/18$ (but entry is not blockaded). Then the uncovered monopoly maximizes, with respect to $P_{IM}$ and $X_{IM}$, profit $\pi_{IM}^{*}=\left(P_{IM}-X_{IM}^2\right)\left(\underline{\theta}+1-P_{IM}/X_{IM}\right)$. Substituting the first-order condition $\partial\pi_{IM}^{*}/\partial P_{IM}=0$ to express the object function in terms of a quality level yields $\pi_{IM}^{*}\left(X_{IM}\right)=X_{IM}\left(\underline{\theta}+1-X_{IM}\right)^2/4$. Now, because $\partial\pi_{IM}^{*}/\partial X_{IM}=\left(\underline{\theta}+1-X_{IM}\right)\left(\underline{\theta}+1-3X_{IM}\right)/4<0$ for all $X_I\in\left((1+\underline{\theta})/3,\hat{X}_I\right)$, $X_I^B=\lambda_H$ would be the incumbent's choice when it decides to deter entry. Also, we already know that if entry occurs with a high quality, $\partial\pi_I^{*}/\partial X_I>0$ for all $X_I\in\left((\underline{\theta}-1)/2,\hat{X}_I\right)$. Thus, $X_I\to\lambda_H$ is the profit-maximizing level of quality when entry is accommodated, so that the maximum payoff that the incumbent can get from the accommodation of entry is $\lim\limits_{X_I\to\lambda_H}\pi_I^{*}\left(X_I, X_E\left(X_I\right)\right)$. Then the incumbent finds it most profitable to deter entry if $\pi_{IM}^{*}\left(X_I^B=\lambda_H\right)>\lim\limits_{X_I\to\lambda_H}\pi_I^{*}\left(X_I, X_E\left(X_I\right)\right)$, or equivalently,

$$J\equiv\frac{3^5}{4}\lambda_H\left(\underline{\theta}+1-\lambda_H\right)^2+\left(2\lambda_H-2-\underline{\theta}\right)\left(4\lambda_H+5-2\underline{\theta}\right)^2>0 \tag{19}$$

Note that because $T>0$ at $F=1/18$ and $\partial T/\partial F>0$, the inequality (19) also holds. Thus, the incumbent will deter entry by choosing $\lambda_H$ as its quality level.[10]

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[10] We assumed that the prospective entrant enters the market only if it can make strictly positive payoffs. If, instead, we were to use a non-negative profit criterion for entry, then we need to distinguish two main cases. When $F=1/18$ the non-negative profit entry criterion yields multiple equilibria (for the incumbent's choice $X_I=\hat{X}_I$ it would be an equilibrium for the entrant to choose a quality level corresponding to either point *b* or point *e* in Figure 3.6 or to decide not to enter the market). When

### *3.3. Summary of Incumbent Strategies*

We now characterize the incumbent's equilibrium qualities that arise in various entry-deterrence strategies faced with potential entry. Market equilibrium values for each entry-deterrence regime are summarized in Table 1. For entry costs such that $F \geq 1/18$, "deterred entry" (DE) or "blockaded entry" (BE) ensure that the potential entrant cannot obtain a positive payoff. In this region of the entry cost, the incumbent may modify its quality-choice behavior relative to the pure monopoly solution in order to prevent entry. The incumbent monopoly market is segmented, as shown in Figure 6.

Whether to deter or accommodate entry depends on the magnitude of entry costs and on the consumer heterogeneity parameter $\underline{\theta}$. First, if the entry cost is sufficiently high, the entrant will not enter even when the incumbent plays its pure monopoly quality level. That is, in this case, the incumbent firm blockades entry simply by choosing its pure monopolist's quality level. Second, for a certain moderate range of entry costs, the unconstrained monopoly optimum cannot be achieved because the pure monopoly equilibrium level of quality is not adequate to deter entry. If the incumbent firm also cannot gain from the differentiated market, in this case the incumbent engages in entry deterrence by increasing its product quality to prevent the prospective entrant from entering the market. Third, when the entry cost is sufficiently low such that $F < 1/18$, entry is accommodated and the incumbent selects a quality that is strictly higher than the monopolist's choice. Note that if entry takes place, the entrant's Stackelberg profits are unchanged regardless of entry qualities.[11] Thus, the entrant's choices are indifferent between

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$F > 1/18$, on the other hand, the non-negative profit entry criterion still yields the same unique Nash equilibrium associated with entry deterrence. For the incumbent's choice $X_I = \lambda_H$ the entrant is now indifferent between entering or not. But if the entrant does enter, then the incumbent has a profitable deviation (by slightly increasing its quality level from $\lambda_H$) and so that cannot be part of a Nash equilibrium. Hence, the choice $X_I = \lambda_H$ would be part of a Nash equilibrium only if the entrant does stay out of the market.

[11] Note also that, there is a first-mover advantage associated with quality leadership: when entry is accommodated, the incumbent (the Stackelberg leader) is in a position to obtain more profits than the entrant (the Stackelberg follower) regardless of the entrant's quality superiority or inferiority (i.e.,

entry with an inferior quality and entry with a superior quality when entry is accommodated by the incumbent. The following Proposition 1 and Figure 7 characterize the entrant's quality choice and the incumbent's deterrence strategies.

***Proposition 1.*** *Fixed entry costs and consumer heterogeneity affect the equilibrium solution as follows: (i) the entrant chooses either low-quality entry or high-quality entry, and the incumbent accommodates this if* $F < 1/18$ *and* $\underline{\theta} \geq \sqrt{19/12}$ *; (ii) entry is deterred if* $F \in \left[1/18\,,\hat{F}\right)$ for $\underline{\theta} \in \left[\sqrt{19/12}\,,2\right]$*, or* $F \in \left[1/18\,,(2/3)^5\right)$ *for* $\underline{\theta} \geq 2$*; (iii) entry is blockaded if* $F \geq \hat{F}$ *for* $\underline{\theta} \in \left[\sqrt{19/12}\,,2\right]$*, or* $F \geq \left(2/3\right)^5$ *for* $\underline{\theta} \geq 2$*, where* $\hat{F} \equiv \left(2/81\right)^2\left(4+\underline{\theta}\right)^3$.

**4. Welfare**

In this section we consider the normative aspects of the entry problem that we have studied. First, we investigate how the market equilibrium level of consumer surplus and social welfare is affected by changes in fixed entry costs. Second, we evaluate the entry-deterrence strategies of the incumbent in terms of social welfare criteria by solving the social planner's maximization problem.

***4.1. Consumer Surplus***

Aggregate consumer surplus, defined as the sum of the surplus of consumers who buy the low-quality good and that of those who buy the high-quality good, is[12]

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$\pi_I > \pi_E$ ). In particular, the first-mover's equilibrium quality does not change whether the accommodated entry accompanies an inferior or a superior quality. Quality differences at either type of accommodated entry equilibrium are the same and equal ½.

[12] As mentioned earlier, subscripts 1 and 2 denote the incumbent firm and the entrant firm, respectively, when entry occurs with a superior quality. The opposite notation applies with the entry of an inferior-quality good.

$$
\begin{aligned}
CS &= \int_{\underline{\theta}}^{\theta_{12}} (\theta X_1 - P_1)\, d\theta + \int_{\theta_{12}}^{1+\underline{\theta}} (\theta X_2 - P_2)\, d\theta \\
&= \frac{(P_2 - P_1)^2}{2(X_2 - X_1)} + \frac{X_2}{2}(1+\underline{\theta})^2 - \frac{X_1}{2}\underline{\theta}^2 + P_1\underline{\theta} - P_2(1+\underline{\theta})
\end{aligned} \quad . \tag{20}
$$

In the absence of entry, consumer surplus associated with the incumbent monopolist's market is[13]

$$
CS_{IM} = \begin{cases} \int_{\frac{P_{IM}}{X_{IM}}}^{1+\underline{\theta}} (\theta X_{IM} - P_{IM})\, d\theta = \frac{(1+\underline{\theta})^2 X_{IM}}{2} - (1+\underline{\theta}) P_{IM} + \frac{P_{IM}^2}{2X_{IM}}, & \text{for uncovered monopoly} \\ \int_{\underline{\theta}}^{1+\underline{\theta}} (\theta X_{IM} - P_{IM})\, d\theta = \frac{(1+2\underline{\theta}) X_{IM}}{2} - P_{IM}, & \text{for covered monopoly} \end{cases} \tag{21}
$$

Substituting market equilibrium values of the quality in Table 1 into these definitions yields consumer surplus for each entry-deterrence regime:

$$
\text{For uncovered monopoly, } CS^* = \begin{cases} \frac{36\underline{\theta}^2 + 36\underline{\theta} - 35}{144}, & \text{if } 0 < F < \frac{1}{18} \\ \frac{8\underline{\theta}^3 + 28\underline{\theta}^2 + 30\underline{\theta} + 9}{512}, & \text{if } F = \frac{1}{18} \\ \frac{\lambda_H^3 - 2(1+\underline{\theta})\lambda_H^2 + (1+\underline{\theta})^2 \lambda_H}{8}, & \text{if } \frac{1}{18} < F < \hat{F} \\ \frac{(1+\underline{\theta})^3}{54}, & \text{if } F \ge \hat{F} \end{cases} \tag{22.1}
$$

$$
\text{For covered monopoly, } CS^* = \begin{cases} \frac{36\underline{\theta}^2 + 36\underline{\theta} - 35}{144}, & \text{if } 0 < F < \frac{1}{18} \\ \frac{2\underline{\theta} + 1}{8}, & \text{if } F = \frac{1}{18} \\ \frac{\lambda_H}{2}, & \text{if } \frac{1}{18} < F < \left(\frac{2}{3}\right)^5 \\ \frac{\underline{\theta}}{4}, & \text{if } F \ge \left(\frac{2}{3}\right)^5 \end{cases} \tag{22.2}
$$

where $\lambda_H \equiv 1 + \frac{\underline{\theta}}{2} - \left(\frac{3}{2}\right)^{5/3} F^{1/3}$ and $\hat{F} \equiv \left(\frac{2}{81}\right)^2 (4+\underline{\theta})^3$.

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[13] Regardless of whether entry is deterred or blockaded, the expression of aggregate consumer surplus is given by equation (21).

Note that, whether the entry quality is superior or inferior compared with the incumbent's quality level, consumer surpluses from the high- and low-quality entry are both equal to $(36\underline{\theta}^2 + 36\underline{\theta} - 35)/144$ when the entry is accommodated.

Figure 8 depicts how consumer surplus changes as the fixed entry cost changes. The response has three distinctive phases. First, when the fixed cost is so large that entry is blockaded, the incumbent's quality choice and its price are not dependent on the magnitude of a fixed cost. Thus, the consumer surplus is constant in this region. Second, when the fixed entry cost decreases and so entry is not blockaded, the incumbent increases its quality level to deter entry as fixed costs decrease. In this case, the consumer surplus of the relatively homogenous group (leading to the covered monopoly case) increases as fixed costs decrease, while relatively heterogeneous consumers (the uncovered monopoly case) become worse off. Third, when the fixed cost is so small that the incumbent cannot deter entry, the consumer surplus is independent of the level of fixed cost because the entrant's positive-profit conditions, which depend on $F$, are not binding. In particular, the consumer surplus from the accommodated entry is higher than that of the deterred entry and blockaded entry. The following proposition summarizes how consumer surplus varies across fixed costs.

***Proposition 2.*** *(i) The consumer surplus for cases with relatively homogeneous consumers is non-increasing in fixed costs. That is, both actual entry and the potential entry associated with the deterred entry increase consumer surplus relative to the pure monopoly situation. (ii) For cases with relatively heterogeneous consumers, the consumer surplus from the accommodated entry is higher than that of the blockaded and deterred entry. The threat of entry associated with the deterred entry, however, makes consumers worse off.*

### *4.2. Equilibrium Social Welfare*

Combining measures of consumer surplus along with firm profits, in the case in which the potential entrant actually enters the market, yields social welfare

$$\begin{aligned} W(X_1, X_2; P_1, P_2) = {} & \frac{(P_2 - P_1)^2}{2(X_2 - X_1)} + \frac{X_2}{2}(1+\underline{\theta})^2 - \frac{X_1}{2}\underline{\theta}^2 + P_1\underline{\theta} - P_2(1+\underline{\theta}) \\ & + \left(P_1 - X_1{}^2\right)\left(\frac{P_2 - P_1}{X_2 - X_1} - \underline{\theta}\right) + \left(P_2 - X_2{}^2\right)\left(1 + \underline{\theta} - \frac{P_2 - P_1}{X_2 - X_1}\right) - F. \end{aligned} \tag{23}$$

In the absence of entry, social welfare is defined by

$$W(X_{IM}; P_{IM}) = \begin{cases} \dfrac{(1+\underline{\theta})^2 X_{IM}}{2} - (1+\underline{\theta})P_{IM} + \dfrac{P_{IM}^2}{2X_{IM}} \\ \qquad + \left(P_{IM} - X_{IM}^2\right)\left(1 + \underline{\theta} - \dfrac{P_{IM}}{X_{IM}}\right), & \text{for uncovered monopoly} \\ \dfrac{(1+2\underline{\theta})X_{IM}}{2} - P_{IM} + \left(P_{IM} - X_{IM}^2\right), & \text{for covered monopoly.} \end{cases} \tag{24}$$

Substituting market equilibrium values of quality from Table 1 into these definitions yields social welfare for each entry-deterrence regime:

$$\text{For } \underline{\theta} \in \left[\sqrt{\frac{19}{12}},\ 2\right],\ W^* = \begin{cases} \dfrac{36\underline{\theta}^2 + 36\underline{\theta} + 5}{144} - F, & \mathit{if}\ 0 < F < \dfrac{1}{18} \\ \dfrac{24\underline{\theta}^3 + 84\underline{\theta}^2 + 90\underline{\theta} + 27}{512}, & \mathit{if}\ F = \dfrac{1}{18} \\ \dfrac{3\lambda_H{}^3 - 6(1+\underline{\theta})\lambda_H{}^2 + 3(1+\underline{\theta})^2\lambda_H}{8}, & \mathit{if}\ \dfrac{1}{18} < F < \hat{F} \\ \dfrac{(1+\underline{\theta})^3}{18}, & \mathit{if}\ F \geq \hat{F} \end{cases} \tag{25.1}$$

$$\text{For } \underline{\theta} \in \left[2,\ 4-2\left(\frac{3}{2}\right)^{\frac{5}{3}} F^{\frac{1}{3}}\right],\ W^* = \begin{cases} \dfrac{36\underline{\theta}^2+36\underline{\theta}+5}{144} - F,\ \text{if } 0<F<\dfrac{1}{18} \\ \dfrac{24\underline{\theta}^3+84\underline{\theta}^2+90\underline{\theta}+27}{512},\ \text{if } F=\dfrac{1}{18} \\ \dfrac{3\lambda_H{}^3-6(1+\underline{\theta})\lambda_H{}^2+3(1+\underline{\theta})^2\lambda_H}{8},\ \text{if } \dfrac{1}{18}<F<\left(\dfrac{2}{3}\right)^5 \\ \dfrac{\underline{\theta}^2+\underline{\theta}}{4},\ \text{if } F\geq\left(\dfrac{2}{3}\right)^5 \end{cases} \tag{25.2}$$

$$\text{For } \underline{\theta} \in \left[4-2\left(\frac{3}{2}\right)^{\frac{5}{3}} F^{\frac{1}{3}},\ \frac{5}{2}\right],\ W^* = \begin{cases} \dfrac{36\underline{\theta}^2+36\underline{\theta}+5}{144} - F,\ \text{if } 0<F<\dfrac{1}{18} \\ \dfrac{24\underline{\theta}^3+84\underline{\theta}^2+90\underline{\theta}+27}{512},\ \text{if } F=\dfrac{1}{18} \\ \lambda_H\left(\dfrac{1}{2}+\underline{\theta}-\lambda_H\right),\ \text{if } \dfrac{1}{18}<F<\left(\dfrac{2}{3}\right)^5 \\ \dfrac{\underline{\theta}^2+\underline{\theta}}{4},\ \text{if } F\geq\left(\dfrac{2}{3}\right)^5 \end{cases} \tag{25.3}$$

$$\text{For } \underline{\theta} \geq \frac{5}{2},\qquad W^* = \begin{cases} \dfrac{36\underline{\theta}^2+36\underline{\theta}+5}{144} - F,\ \text{if } 0<F<\dfrac{1}{18} \\ \left(\dfrac{2\underline{\theta}+1}{4}\right)^2,\ \text{if } F=\dfrac{1}{18} \\ \lambda_H\left(\dfrac{1}{2}+\underline{\theta}-\lambda_H\right),\ \text{if } \dfrac{1}{18}<F<\left(\dfrac{2}{3}\right)^5 \\ \dfrac{\underline{\theta}^2+\underline{\theta}}{4},\ \text{if } F\geq\left(\dfrac{2}{3}\right)^5 \end{cases} \tag{25.4}$$

where $\lambda_H \equiv 1+\dfrac{\underline{\theta}}{2}-\left(\dfrac{3}{2}\right)^{5/3} F^{1/3}$ and $\hat{F} \equiv \left(\dfrac{2}{81}\right)^2 (4+\underline{\theta})^3$.

Figure 9 depicts how the market equilibrium level of social welfare changes as the fixed entry cost changes. The total welfare of accommodated entry depends on the fixed entry cost. As we can see, maximum welfare is not necessarily associated with the case of accommodated entry. Although it is deterred, potential entry may be welfare enhancing relative to the pure monopoly situation. Thus, the following proposition holds.

***Proposition 3.*** *Entry deterrence is not necessarily welfare decreasing. For the case of relatively homogeneous consumers, maximum welfare is attained at* $F = 1/18$*, where entry is deterred. For relatively heterogeneous consumers, maximum welfare is attained at* $F = 0$*, where entry is accommodated.*

### *4.3. Social Optimum*

If a social planner were to introduce a new variety, the planner would determine the socially optimal level of qualities under marginal cost pricing. We suppose that the planner also needs a fixed entry cost to choose a new variety while she does not need it to choose an existing variety. We assume that this fixed cost is the same as the entry cost $F$. Thus, in the presence of entry, the planner maximizes the sum of profits and consumer surplus as

$$\begin{aligned} &\underset{X_1,\, X_2}{Max} \;\; W = \int_{\underline{\theta}}^{\tilde{\theta}_{12}} \left(\theta X_1 - X_1^{\;2}\right) d\theta + \int_{\tilde{\theta}_{12}}^{1+\underline{\theta}} \left(\theta X_2 - X_2^{\;2}\right) d\theta - F \\ &s.t. \qquad \tilde{\theta}_{12} = \frac{\tilde{P}_2 - \tilde{P}_1}{X_2 - X_1} = \frac{X_2^{\;2} - X_1^{\;2}}{X_2 - X_1} = X_2 + X_1. \end{aligned} \tag{26}$$

Solving the problem in (26) yields the efficient level of qualities as $\tilde{X}_1 = \underline{\theta}/2 + 1/8$ and $\tilde{X}_2 = \underline{\theta}/2 + 3/8$. Note that, in our parameter ranges on $\underline{\theta}$, the market will be fully covered with these optimal qualities because $\underline{\theta} \geq \tilde{P}_1/\tilde{X}_1 = \tilde{X}_1^2/\tilde{X}_1 = \tilde{X}_1 = \underline{\theta}/2 + 1/8 \iff \underline{\theta} \geq 1/4$. Meanwhile, if the planner decides not to introduce a new variety in the economy, then the optimal quality is determined by solving

$$\underset{X}{Max} \;\; W = \int_{\underline{\theta}}^{1+\underline{\theta}} \left(\theta X - X^2\right) d\theta . \tag{27}$$

Straightforward calculation yields $\tilde{X} = \underline{\theta}/2 + 1/4$. Note that, if our parameter ranges on $\underline{\theta}$, the market will be fully covered with $\tilde{X}$ because $\underline{\theta} \geq \tilde{P}/\tilde{X} = \tilde{X} = \underline{\theta}/2 + 1/4 \iff \underline{\theta} \geq 1/2$.

If the planner accommodates a new variety in the economy, $W\left(\tilde{X}_1, \tilde{X}_2\right) - F = \left(16\underline{\theta}^2 + 16\underline{\theta} + 5\right)/64 - F$. If only one variety is allowed in the economy,

$W\left(\tilde{X}\right)=\left(16\underline{\theta}^2+16\underline{\theta}+4\right)\big/64$. Thus, the planner accommodates a new variety in the economy if

$W\left(\tilde{X}_1,\tilde{X}_2\right)-F>W\left(\tilde{X}\right)$, i.e., whenever $F<1/64$.

Now, let us compare the market equilibrium level of qualities to the socially optimal level of qualities. In the absence of entry, $X_{IM}^{*}=(1+\underline{\theta})/3<(\underline{\theta}/2+1/4\ )=\tilde{X}$ for $\underline{\theta}\in\left[\sqrt{19/12}\ ,2\right]$, $X_{IM}^{*}=\underline{\theta}/2<\underline{\theta}/2+1/4=\tilde{X}$ for $\underline{\theta}\geq 2$, and $X_I^B=\lambda_H<\underline{\theta}/2+1/4=\tilde{X}$. When entry is accommodated, therefore, profit maximization yields a quality difference that is too high; i.e., $\left(\tilde{X}_2-\tilde{X}_1\right)-\left(X_2{}^{*}-X_1{}^{*}\right)=1/4-1/2<0$. Then the following proposition summarizes these results.

***Proposition 4.*** *(i) The level of entry costs that makes it socially optimal to have a new quality of good in the economy is* $F<1/64$. *Thus, for* $F\in\left[1/64\ ,1/18\right)$*, there are too many varieties in the economy relative to the social optimum. (ii) For a fixed entry cost with* $F<1/64$, *Stackelberg firms provide excessive product differentiation, compared with what would be socially desirable. (iii) The incumbent monopolist, whether the entry is deterred or blockaded, strictly undersupplies product quality relative to the social optimum.*

## 5. Conclusion

We have analyzed the strategic use of entry deterrence of an established firm, and the entrant's quality choice, in a vertically differentiated product market. We have characterized the equilibrium properties of the three-stage game in which quality choice is sequential, price competition occurs at the last stage, production costs are quality-dependent, and a fixed entry cost is required to the potential entrant firm. With the simplest case of one incumbent firm facing one prospective entrant, we showed how the incumbent's pre-entry decision generates various equilibrium qualities. In our Stackelberg game, the incumbent influences the quality choice of the entrant by choosing its quality level before the entrant does. This allows the incumbent to limit the entrant's entry decision and quality levels. We characterized the levels of the entrant's fixed costs, and the degree of consumer homogeneity, that induce the incumbent to engage, in equilibrium, in either entry deterrence or entry accommodation. Also, we compared market equilibrium values to the socially optimal ones.

We find that, first, when the entrant's fixed cost is sufficiently low, the incumbent's optimal strategy is to accommodate entry, and the entrant's choices are indifferent between entry with an inferior quality and entry with a superior quality. In this case, the incumbent selects a quality that is higher than the monopolist's choice. Second, if the entry cost is in a certain moderate range, the incumbent engages in entry deterrence by increasing its product quality before the entrant enters the market. Third, for a sufficiently high fixed entry cost, entry is efficiently blockaded and the incumbent chooses the monopolist's quality level. Fourth, it is shown that while consumer surplus is higher when the entry is accommodated than in the absence of entry, maximum total welfare is not necessarily associated with the accommodated entry. In particular, the maximum welfare for the case of relatively homogenous consumers is attained at the fixed cost level where entry would be deterred. Fifth, for a certain level of fixed entry costs, there are too many varieties in the economy relative to the social optimum. We also show that Stackelberg firms associated with accommodated entry excessively differentiate product qualities to reduce price competition. The incumbent monopolist, whether the entry is deterred or blockaded, strictly undersupplies product quality relative to the social optimum.

We again stress that our analysis on how the existence of a potential entrant influences quality relies on a VPD model with the assumption of quality-dependent variable costs. With this quality-cost specification, as mentioned earlier, the "high-quality advantage" does not necessarily hold. Actually, we have shown that the incumbent's profit is greater than the entrant's profit, regardless of the entrant's quality regime (i.e., there is a first-mover advantage).

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**Table 1. Comparison of Equilibrium Values of Entry-Deterrence Regimes**

| Type of Entry | Variables | Uncovered Monopoly | Covered Monopoly |
|---|---|---|---|
| Blockaded Entry | Conditions on $(F,\ \underline{\theta})$ | When $F \geq \hat{F}$ and $\sqrt{\frac{19}{12}} \leq \underline{\theta} \leq 2$, where $\hat{F} \equiv \left(\frac{2}{81}\right)^2 (4+\underline{\theta})^3$ | When $F \geq \left(\frac{2}{3}\right)^5$ and $\underline{\theta} \geq 2$ |
| | $X_I$ | $\frac{1+\underline{\theta}}{3}$ | $\frac{\underline{\theta}}{2}$ |
| | $P_I$ | $\frac{2(1+\underline{\theta})^2}{9}$ | $\frac{\underline{\theta}^2}{2}$ |
| | $\pi_I$ | $\left(\frac{1+\underline{\theta}}{3}\right)^3$ | $\left(\frac{\underline{\theta}}{2}\right)^2$ |
| Deterred Entry | Conditions on $(F,\ \underline{\theta})$ | When $\frac{1}{18} < F < \hat{F}$ for $\underline{\theta} \in \left[\sqrt{\frac{19}{12}},\ 2\right]$, $\frac{1}{18} < F < \left(\frac{2}{3}\right)^5$ for $\underline{\theta} \in \left[2,\ 4-2\left(\frac{3}{2}\right)^{\frac{5}{3}} F^{\frac{1}{3}}\right]$, and $\sqrt{\frac{19}{12}} \leq \underline{\theta} \leq 4-2\left(\frac{3}{2}\right)^{\frac{5}{3}} F^{\frac{1}{3}}$ | When $\frac{1}{18} < F < \left(\frac{2}{3}\right)^5$ and $\underline{\theta} \geq 4-2\left(\frac{3}{2}\right)^{\frac{5}{3}} F^{\frac{1}{3}}$ |
| | $X_I$ | $\lambda_H = 1+\frac{\underline{\theta}}{2}-\left(\frac{3}{2}\right)^{5/3} F^{1/3}$ | $\lambda_H = 1+\frac{\underline{\theta}}{2}-\left(\frac{3}{2}\right)^{5/3} F^{1/3}$ |
| | $P_I$ | $\frac{\lambda_H(1+\underline{\theta}+\lambda_H)}{2}$ | $\underline{\theta}\lambda_H$ |
| | $\pi_I$ | $\lambda_H\left(\frac{1+\underline{\theta}-\lambda_H}{2}\right)^2$ | $\underline{\theta}\lambda_H - \lambda_H^2$ |
| | Conditions on $(F,\ \underline{\theta})$ | When $F = \frac{1}{18}$ and $\sqrt{\frac{19}{12}} \leq \underline{\theta} \leq \frac{5}{2}$ | When $F = \frac{1}{18}$ and $\underline{\theta} \geq \frac{5}{2}$ |
| | $X_I$ | $\frac{\underline{\theta}}{2}+\frac{1}{4}$ | $\frac{\underline{\theta}}{2}+\frac{1}{4}$ |
| | $P_I$ | $\frac{(2\underline{\theta}+1)(6\underline{\theta}+5)}{32}$ | $\frac{\underline{\theta}(2\underline{\theta}+1)}{4}$ |
| | $\pi_I$ | $\frac{(2\underline{\theta}+1)(2\underline{\theta}+3)^2}{256}$ | $\frac{(2\underline{\theta}+1)(2\underline{\theta}-1)}{16}$ |
| Accommodated Entry | Conditions on $(F,\ \underline{\theta})$ | When $0 < F < \frac{1}{18}$ and $\underline{\theta} \geq \sqrt{\frac{19}{12}}$ | |
| | $(X_I,\ X_E)$ | $\left(\frac{\underline{\theta}}{2}+\frac{1}{4},\ \frac{\underline{\theta}}{2}+\frac{3}{4}\right)$ or $\left(\frac{\underline{\theta}}{2}+\frac{1}{4},\ \frac{\underline{\theta}}{2}-\frac{1}{4}\right)$ | |
| | $(P_I,\ P_E)$ | $\left(\frac{12\underline{\theta}^2+12\underline{\theta}+19}{48},\ \frac{12\underline{\theta}^2+36\underline{\theta}+35}{48}\right)$ when $X_E = \frac{\underline{\theta}}{2}+\frac{3}{4}$ <br> $\left(\frac{12\underline{\theta}^2+12\underline{\theta}+19}{48},\ \frac{12\underline{\theta}^2-12\underline{\theta}+11}{48}\right)$ when $X_E = \frac{\underline{\theta}}{2}-\frac{1}{4}$ | |
| | $(\pi_I,\ \pi_E)$ | $\left(\frac{2}{9},\ \frac{1}{18}-F\right)$ | |

**Figure 1. Post-innovative Market Structure with a Covered Market**

(Relative consumer heterogeneity decreases as $\underline{\theta}$ increases)

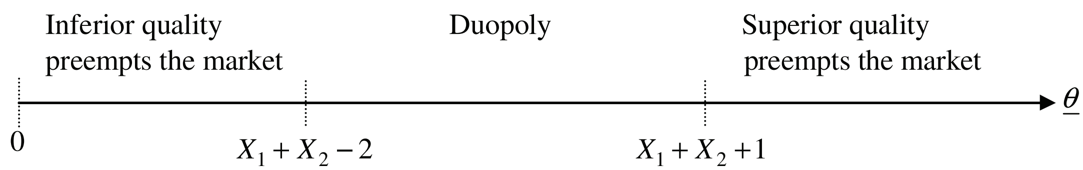


**Figure 2. The Best-Response Function of the Entrant (when $F < 1/18$)**

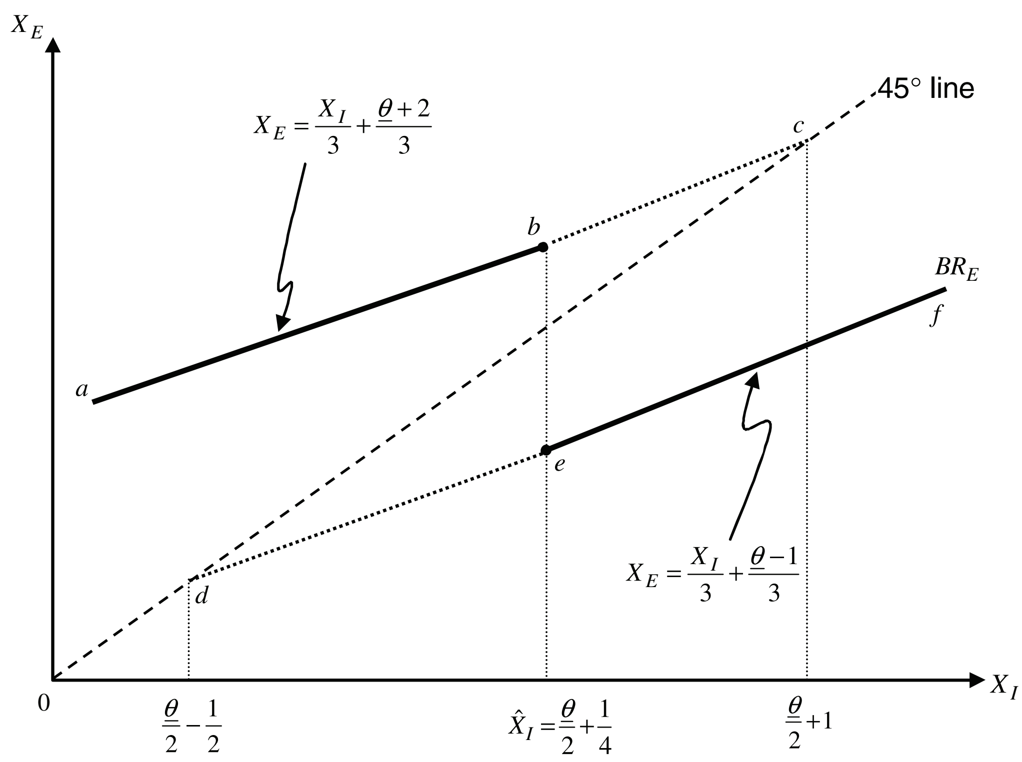

**Figure 3. The Best-Response Function of the Entrant (when $F = 1/18$)**

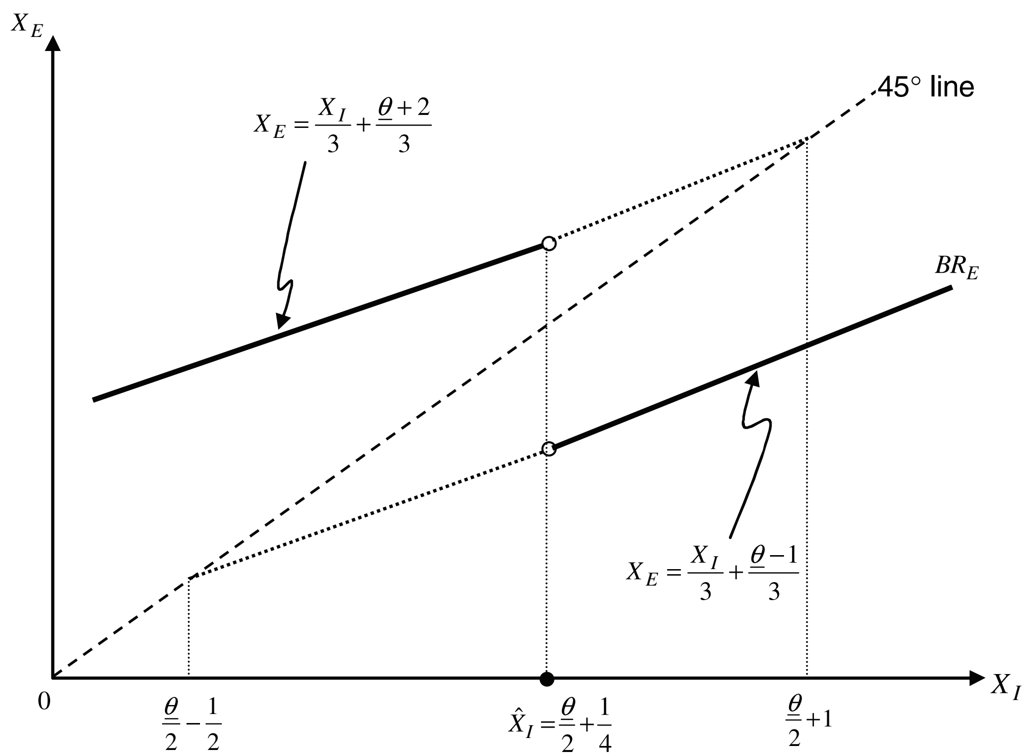

**Figure 4. The Best-Response Function of the Entrant (when $F > 1/18$)**

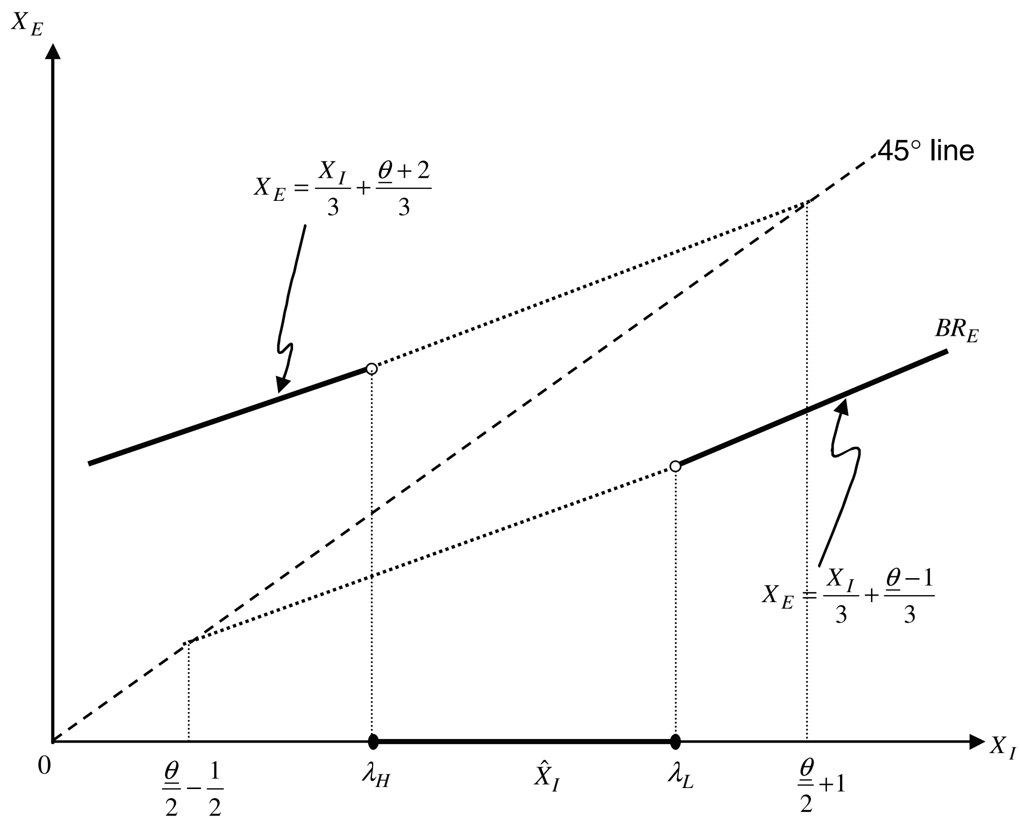

**Figure 5. Equilibrium Market Segmentation**

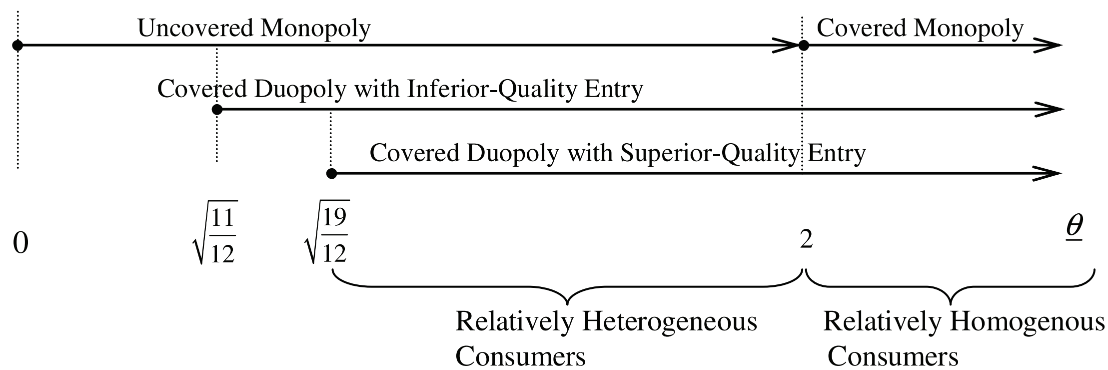


**Figure 6. Monopoly Market Segmentation**

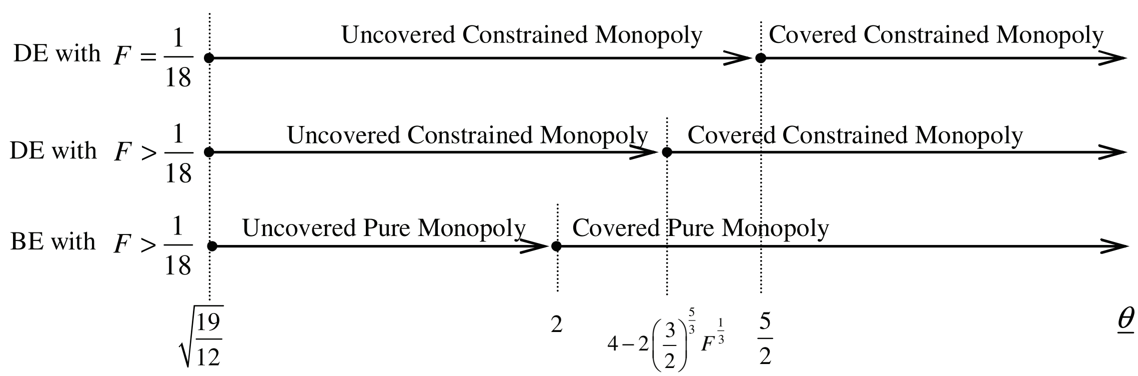

**Figure 7. Zones of a Strategic Entry and Entry-Deterrence Decision**

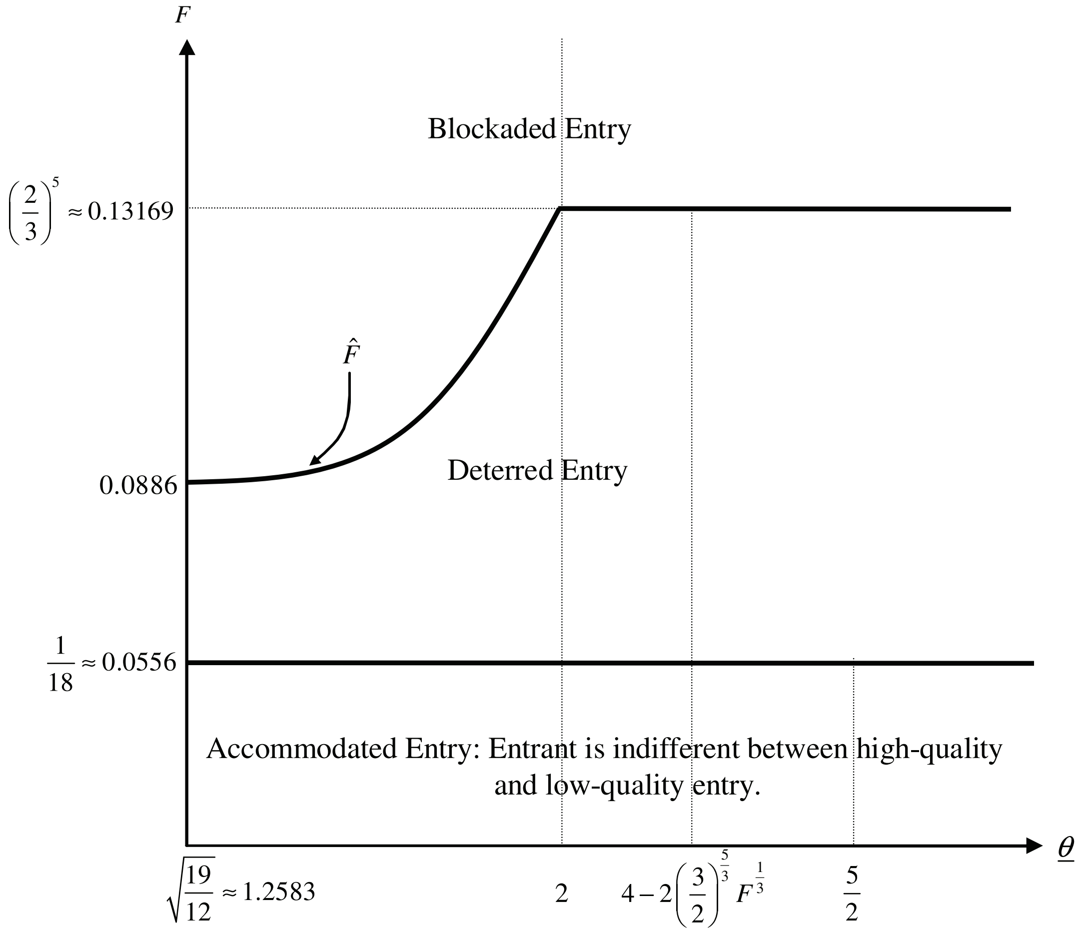

**Figure 8-a. Consumer Surplus: Uncovered Monopoly**

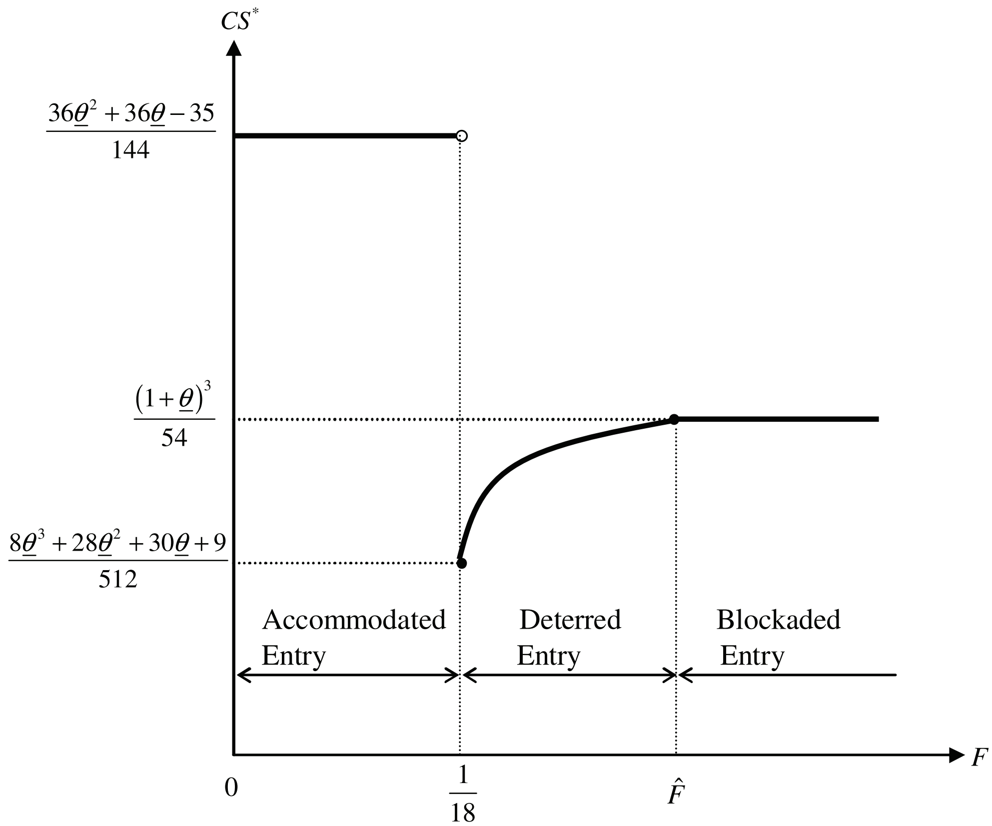


**Figure 8-b. Consumer Surplus: Covered Monopoly**

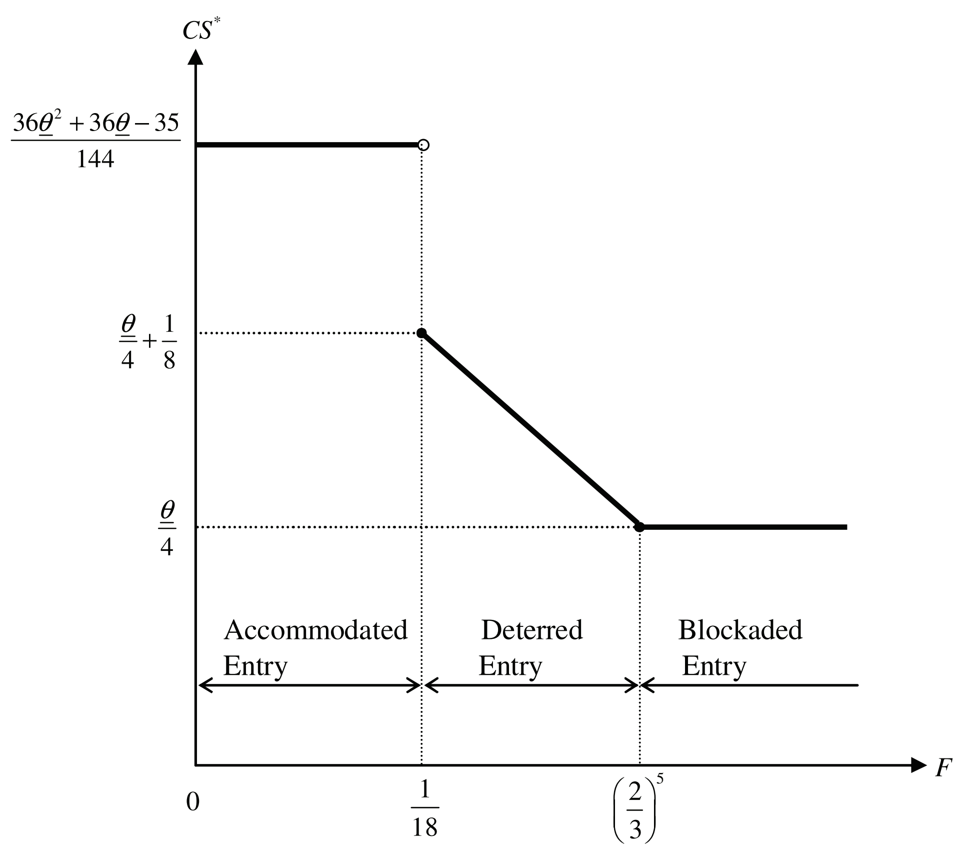

**Figure 9. Equilibrium Social Welfare**

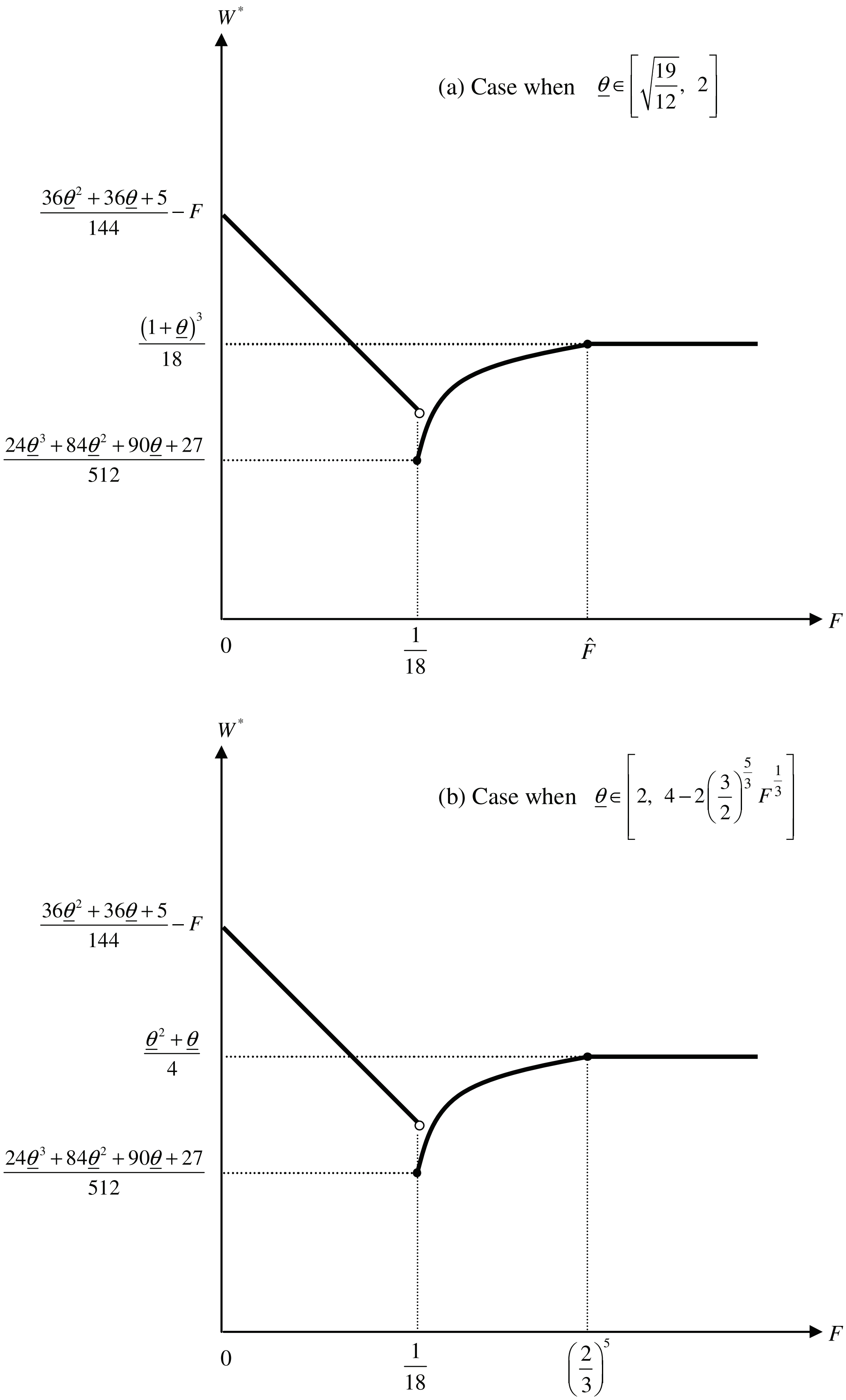

**Figure 9 (continued). Equilibrium Social Welfare**